

# Contents

Chapter 1. Introduction	1
Chapter 2. Notation	9
Chapter 3. Preliminaries	13
Chapter 4. Strategy for the proofs of Theorems 5.1–9.1	21
Chapter 5. Irreducible subgroups of $G_2$	25
Chapter 6. Irreducible subgroups of $F_4$	27
6.1. $M = B_4 (F_4(\#12))$	27
6.2. $M = C_4 (p = 2) (F_4(\#14))$	28
6.3. $M = \bar{A}_1 C_3 (F_4(\#24))$	29
6.4. $M = A_1 G_2 (p \neq 2) (F_4(\#25))$	29
6.5. $M = \bar{A}_2 \bar{A}_2 (F_4(\#26))$	29
6.6. $M = G_2 (p = 7) (F_4(\#16))$	29
Chapter 7. Irreducible subgroups of $G = E_6$	31
7.1. $M = \bar{A}_1 A_5 (E_6(\#24))$	31
7.2. $M = \bar{A}_2^3 (E_6(\#25))$	32
7.3. $M = A_2 G_2 (E_6(\#26))$	33
7.4. $M = F_4 (E_6(\#7))$	33
7.5. $M = C_4 (E_6(\#8))$	33
7.6. $M = G_2 (E_6(\#10))$	34
Chapter 8. Irreducible subgroups of $G = E_7$	35
8.1. $M = \bar{A}_1 D_6 (E_7(\#30))$	35
8.2. $M = \bar{A}_2 A_5 (E_7(\#31))$	40
8.3. $M = A_7 (E_7(\#22))$	42
8.4. $M = G_2 C_3 (E_7(\#32))$	42
8.5. $M = A_1 F_4 (E_7(\#33))$	43
8.6. $M = A_1 G_2 (p \neq 2) (E_7(\#34))$	43
Chapter 9. Irreducible subgroups of $G = E_8$	45
9.1. $M = D_8 (E_8(\#43))$	45
9.2. $M = \bar{A}_1 E_7 (E_8(\#102))$	51
9.3. $M = \bar{A}_2 E_6 (E_8(\#103))$	53
9.4. $M = A_8 (E_8(\#62))$	55
9.5. $M = \bar{A}_4^2 (E_8(\#104))$	55
9.6. $M = G_2 F_4 (E_8(\#105))$	55

9.7.	$M = F_4 (p = 3) (E_8(\#1049))$	56
9.8.	$M = B_2 (p \geq 5) (E_8(\#101))$	57
9.9.	$M = A_1A_2 (p \geq 5) (E_8(\#106))$	57
Chapter 10.	Corollaries	59
10.1.	Variations of Steinberg's Tensor Product Theorem	62
Chapter 11.	Tables for Theorem 1	65
11.1.	Irreducible diagonal subgroups	130
Chapter 12.	Composition factors for $G$ -irreducible subgroups	157
Chapter 13.	Composition factors for the action of Levi subgroups	183
	Bibliography	189