

Contents

| | | |
|------------------|---|------------|
| | Preface | v |
| CHAPTER 1 | Basic Concepts | 1 |
| | 1.1 Simple Differential Equations and Explicit Solutions | 1 |
| | 1.2 Graphical Solutions Using Calculus | 8 |
| | 1.3 Slope Fields and Isoclines | 12 |
| | 1.4 Functions and Power Series Expansions | 26 |
| CHAPTER 2 | Autonomous Differential Equations | 31 |
| | 2.1 Autonomous Equations | 32 |
| | 2.2 Simple Applications | 44 |
| | 2.3 The Logistics Equation | 53 |
| | 2.4 Existence and Uniqueness of Solutions and More Words of Caution | 69 |
| | 2.5 Phase Line Analysis for Autonomous Equations | 85 |
| CHAPTER 3 | General First Order Differential Equations | 93 |
| | 3.1 Graphical Solutions | 93 |
| | 3.2 Symmetry | 112 |
| | 3.3 Numerical Solutions: Euler's Method | 119 |
| | 3.4 Power Series Solutions | 126 |
| CHAPTER 4 | Separable Differential Equations and Applications | 131 |
| | 4.1 Separable Equations | 131 |
| | 4.2 Graphing Separable Equations | 146 |
| | 4.3 Applications—Deriving Differential Equations From Data | 151 |
| | 4.4 Applications—Mechanics | 167 |
| | 4.5 Applications—Orthogonal Trajectories | 179 |
| | 4.6 Applications—Functional Equations | 189 |
| | 4.7 Applications—Calculus | 193 |
| | 4.8 Other Applications | 200 |
| CHAPTER 5 | Applications Leading to New Techniques | 209 |
| | 5.1 Differential Equations with Homogeneous Coefficients | 209 |

| | | | |
|-------------------|------|---|------------|
| | 5.2 | Linear Differential Equations | 220 |
| | 5.3 | Bernoulli's Equation | 234 |
| | 5.4 | Clairaut's Equation | 246 |
| | 5.5 | Equations Reducible to First Order | 254 |
| CHAPTER 6 | | First Order Linear Differential Equations and Applications | 267 |
| | 6.1 | Further Analysis of Linear Equations | 267 |
| | 6.2 | The Principle of Linear Superposition | 278 |
| | 6.3 | Solving Linear Differential Equations with Constant Coefficients | 287 |
| | 6.4 | More Applications | 302 |
| CHAPTER 7 | | Second Order Differential Equations with Constant Coefficients | 319 |
| | 7.1 | Examples of Second Order Equations with Constant Coefficients | 319 |
| | 7.2 | General Second Order Linear Differential Equations with Constant Coefficients | 337 |
| | 7.3 | Initial Value Problems and an Existence Theorem | 347 |
| | 7.4 | Constructing the Phase Plane Trajectory from the Explicit Solution | 354 |
| CHAPTER 8 | | Applications | 365 |
| | 8.1 | The Simple Pendulum | 365 |
| | 8.2 | Spring-Mass System | 384 |
| | 8.3 | Electrical Circuits | 405 |
| | 8.4 | Boundary Value Problems | 420 |
| CHAPTER 9 | | Second Order Linear Differential Equations | 431 |
| | 9.1 | General Properties | 431 |
| | 9.2 | Reduction of Order | 439 |
| | 9.3 | Linear Independence and Dependence | 443 |
| | 9.4 | Cauchy-Euler Equation | 450 |
| | 9.5 | Taylor Series | 455 |
| | 9.6 | Power Series and Ordinary Points | 464 |
| CHAPTER 10 | | Generalized Power Series Solutions | 485 |
| | 10.1 | Regular Singular Points | 485 |
| | 10.2 | The Method of Frobenius, Part 1 | 495 |
| | 10.3 | The Method of Frobenius, Part 2 | 503 |
| CHAPTER 11 | | Nonhomogeneous Second Order Linear Differential Equations | 517 |
| | 11.1 | The General Solution | 517 |
| | 11.2 | Method of Undetermined Coefficients | 521 |
| | 11.3 | Reduction of Order and Variation of Parameters | 531 |
| | 11.4 | Applications | 541 |
| | 11.5 | Higher Order Homogeneous Differential Equations | 556 |
| | 11.6 | Higher Order Nonhomogeneous Differential Equations | 568 |
| CHAPTER 12 | | Autonomous Systems | 577 |
| | 12.1 | Solutions of Linear Autonomous Systems | 577 |
| | 12.2 | Stability of Linear Autonomous Systems | 584 |

| | | | |
|-------------------|------|---|------------|
| | 12.3 | Straight Line Trajectories of Linear Autonomous Systems | 599 |
| | 12.4 | Nullcline Analysis of Linear Autonomous Systems | 603 |
| | 12.5 | Nonlinear Autonomous Systems | 610 |
| CHAPTER 13 | | Systems of Linear Differential Equations | 627 |
| | 13.1 | Matrix Formulation of Solutions for Autonomous Systems | 627 |
| | 13.2 | Nonhomogeneous Systems of First Order Linear Differential Equations | 644 |
| | 13.3 | Variation of Parameters | 657 |
| | 13.4 | Applications | 661 |
| | 13.5 | Higher Order Systems | 673 |
| CHAPTER 14 | | The Laplace Transform | 685 |
| | 14.1 | Introduction | 685 |
| | 14.2 | Constructing New Laplace Transforms from Old | 695 |
| | 14.3 | The Inverse Laplace Transform and the Convolution Theorem | 703 |
| | 14.4 | The Unit Step Function | 710 |
| | 14.5 | Applications to First Order Linear Differential Equations | 728 |
| | 14.6 | Applications to Higher Order Linear Differential Equations | 737 |
| | 14.7 | Applications to Systems of Linear Differential Equations | 745 |
| | 14.8 | When Do Laplace Transforms Exist? | 752 |
| | | Appendices | 761 |
| | A.1 | Background Material | 761 |
| | A.2 | Partial Fractions | 768 |
| | A.3 | Infinite Series, Power Series, and Taylor Series | 770 |
| | A.4 | Complex Numbers | 772 |
| | A.5 | Elementary Matrix Operations | 775 |
| | A.6 | Least Squares Approximation | 781 |
| | A.7 | Numerical Techniques for Solving Differential Equations | 781 |
| | A.8 | Proof of Comparison Theorems | 783 |
| | | Index | 787 |