
Contents

Preface to the Second Edition	xiii
Preface to the First Edition	xv
Notation for Special Functions	xvii

Chapter 1. Infinite Series, Improper Integrals, and Infinite Products	1
1.1 Introduction	1
1.2 Infinite Series of Constants	2
1.2.1 The Geometric Series	4
1.2.2 Summary of Convergence Tests	6
1.2.3 Operations with Series	11
1.2.4 Factorials and Binomial Coefficients	15
1.3 Infinite Series of Functions	21
1.3.1 Properties of Uniformly Convergent Series	23
1.3.2 Power Series	25
1.3.3 Sums and Products of Power Series	29
1.4 Fourier Trigonometric Series	33
1.4.1 Cosine and Sine Series	36
1.5 Improper Integrals	39
1.5.1 Types of Improper Integrals	39
1.5.2 Convergence Tests	42
1.5.3 Pointwise and Uniform Convergence	43
1.6 Asymptotic Formulas	47
1.6.1 Small Arguments	48
1.6.2 Large Arguments	50
1.7 Infinite Products	55
1.7.1 Associated Infinite Series	56
1.7.2 Products of Functions	57

Chapter 2. The Gamma Function and Related Functions	61
2.1 Introduction	61
2.2 Gamma Function	62
2.2.1 Integral Representations	64
2.2.2 Legendre Duplication Formula	70
2.2.3 Weierstrass' Infinite Product	71

2.3	Applications	77
2.3.1	Miscellaneous Problems	77
2.3.2	Fractional-Order Derivatives	79
2.4	Beta Function	82
2.5	Incomplete Gamma Function	87
2.5.1	Asymptotic Series	88
2.6	Digamma and Polygamma Functions	90
2.6.1	Integral Representations	93
2.6.2	Asymptotic Series	95
2.6.3	Polygamma Functions	100
2.6.4	Riemann Zeta Function	102
Chapter 3. Other Functions Defined by Integrals		109
3.1	Introduction	109
3.2	Error Function and Related Functions	110
3.2.1	Asymptotic Series	112
3.2.2	Fresnel Integrals	113
3.3	Applications	118
3.3.1	Probability and Statistics	118
3.3.2	Heat Conduction in Solids	119
3.3.3	Vibrating Beams	122
3.4	Exponential Integral and Related Functions	126
3.4.1	Logarithmic Integral	128
3.4.2	Sine and Cosine Integrals	129
3.5	Elliptic Integrals	133
3.5.1	Limiting Values and Series Representations	134
3.5.2	The Pendulum Problem	135
Chapter 4. Legendre Polynomials and Related Functions		141
4.1	Introduction	141
4.2	Legendre Polynomials	142
4.2.1	The Generating Function	142
4.2.2	Special Values and Recurrence Formulas	146
4.2.3	Legendre's Differential Equation	151
4.3	Other Representations of the Legendre Polynomials	157
4.3.1	Rodrigues' Formula	157
4.3.2	Laplace Integral Formula	158
4.3.3	Some Bounds on $P_n(x)$	159
4.4	Legendre Series	162
4.4.1	Orthogonality of the Polynomials	162
4.4.2	Finite Legendre Series	165
4.4.3	Infinite Legendre Series	167
4.5	Convergence of the Series	173
4.5.1	Piecewise Continuous and Piecewise Smooth Functions	174
4.5.2	Pointwise Convergence	175
4.6	Legendre Functions of the Second Kind	181
4.6.1	Basic Properties	184
4.7	Associated Legendre Functions	186
4.7.1	Basic Properties of $P_n^m(x)$	189

4.8	Applications	192
4.8.1	Electric Potential due to a Sphere	193
4.8.2	Steady-State Temperatures in a Sphere	197
Chapter 5. Other Orthogonal Polynomials		203
5.1	Introduction	203
5.2	Hermite Polynomials	204
5.2.1	Recurrence Formulas	206
5.2.2	Hermite Series	207
5.2.3	Simple Harmonic Oscillator	209
5.3	Laguerre Polynomials	214
5.3.1	Recurrence Formulas	215
5.3.2	Laguerre Series	217
5.3.3	Associated Laguerre Polynomials	218
5.3.4	The Hydrogen Atom	221
5.4	Generalized Polynomial Sets	226
5.4.1	Gegenbauer Polynomials	226
5.4.2	Chebyshev Polynomials	228
5.4.3	Jacobi Polynomials	231
Chapter 6. Bessel Functions		237
6.1	Introduction	237
6.2	Bessel Functions of the First Kind	238
6.2.1	The Generating Function	238
6.2.2	Bessel Functions of Nonintegral Order	240
6.2.3	Recurrence Formulas	242
6.2.4	Bessel's Differential Equation	243
6.3	Integral Representations	248
6.3.1	Bessel's Problem	250
6.3.2	Geometric Problems	253
6.4	Integrals of Bessel Functions	256
6.4.1	Indefinite Integrals	256
6.4.2	Definite Integrals	258
6.5	Series Involving Bessel Functions	265
6.5.1	Addition Formulas	265
6.5.2	Orthogonality of Bessel Functions	267
6.5.3	Fourier-Bessel Series	269
6.6	Bessel Functions of the Second Kind	273
6.6.1	Series Expansion for $Y_n(x)$	274
6.6.2	Asymptotic Formulas for Small Arguments	277
6.6.3	Recurrence Formulas	278
6.7	Differential Equations Related to Bessel's Equation	280
6.7.1	The Oscillating Chain	282
Chapter 7. Bessel Functions of Other Kinds		287
7.1	Introduction	287
7.2	Modified Bessel Functions	287
7.2.1	Modified Bessel Functions of the Second Kind	290

7.2.2	Recurrence Formulas	291
7.2.3	Generating Function and Addition Theorems	292
7.3	Integral Relations	298
7.3.1	Integral Representations	298
7.3.2	Integrals of Modified Bessel Functions	299
7.4	Spherical Bessel Functions	302
7.4.1	Recurrence Formulas	305
7.4.2	Modified Spherical Bessel Functions	305
7.5	Other Bessel Functions	308
7.5.1	Hankel Functions	308
7.5.2	Struve Functions	309
7.5.3	Kelvin's Functions	311
7.5.4	Airy Functions	312
7.6	Asymptotic Formulas	316
7.6.1	Small Arguments	316
7.6.2	Large Arguments	317
 Chapter 8. Applications Involving Bessel Functions		 323
8.1	Introduction	323
8.2	Problems in Mechanics	323
8.2.1	The Lengthening Pendulum	323
8.2.2	Buckling of a Long Column	327
8.3	Statistical Communication Theory	332
8.3.1	Narrowband Noise and Envelope Detection	333
8.3.2	Non-Rayleigh Radar Sea Clutter	336
8.4	Heat Conduction and Vibration Phenomena	339
8.4.1	Radial Symmetric Problems Involving Circles	340
8.4.2	Radial Symmetric Problems Involving Cylinders	343
8.4.3	The Helmholtz Equation	345
8.5	Step-Index Optical Fibers	351
 Chapter 9. The Hypergeometric Function		 357
9.1	Introduction	357
9.2	The Pochhammer Symbol	358
9.3	The Function $F(a, b; c; x)$	361
9.3.1	Elementary Properties	362
9.3.2	Integral Representation	364
9.3.3	The Hypergeometric Equation	365
9.4	Relation to Other Functions	370
9.4.1	Legendre Functions	373
9.5	Summing Series and Evaluating Integrals	377
9.5.1	Action-Angle Variables	380
 Chapter 10. The Confluent Hypergeometric Functions		 385
10.1	Introduction	385
10.2	The Functions $M(a; c; x)$ and $U(a; c; x)$	386

10.2.1	Elementary Properties of $M(a; c; x)$	386
10.2.2	Confluent Hypergeometric Equation and $U(a; c; x)$	388
10.2.3	Asymptotic Formulas	390
10.3	Relation to Other Functions	395
10.3.1	Hermite Functions	397
10.3.2	Laguerre Functions	399
10.4	Whittaker Functions	403
Chapter 11.	Generalized Hypergeometric Functions	411
11.1	Introduction	411
11.2	The Set of Functions ${}_pF_q$	412
11.2.1	Hypergeometric-Type Series	413
11.3	Other Generalizations	419
11.3.1	The Meijer G Function	419
11.3.2	The MacRobert E Function	425
Chapter 12.	Applications Involving Hypergeometric-Type Functions	429
12.1	Introduction	429
12.2	Statistical Communication Theory	429
12.2.1	Nonlinear Devices	431
12.3	Fluid Mechanics	437
12.3.1	Unsteady Hydrodynamic Flow Past an Infinite Plate	437
12.3.2	Transonic Flow and the Euler-Tricomi Equation	440
12.4	Random Fields	444
12.4.1	Structure Function of Temperature	445
	Bibliography	451
	Appendix: A List of Special Function Formulas	453
	Selected Answers to Exercises	469
Index	473	